In most radio frequency work it is important to obtain a large ratio of reactance to resistance in the reactive elements of the circuit. This ratio is called the Q of the circuit.

\[ Q = \frac{X_L}{R} = \frac{2\pi f L}{R} = \frac{X_C}{R} = \frac{1}{2\pi f CR} \]

A high Q is required to obtain good efficiency, good waveform, good frequency stability, high gain, etc.

The Q meter is an instrument that measures the Q of a reactance element directly. It may also be used to measure:

- the reactance and resistance of a circuit,
- the distributed capacity of a circuit,
- the resonant frequency of a tuned circuit, etc.

Fig. 4–32 shows the fundamental circuit of the Q meter.
The instrument consists of the variable frequency oscillator, the thermocouple ammeter \( I \), the standard resistor \( R' \), the standard condenser \( C \), and the vacuum-tube voltmeter \( E_C \).

In the Boonton Type 160A the frequency range is 50—75 megacycles; \( C \) has a range of 30—450 microfarads; the \( Q \) range is 50 to 600. In the Type 170A the frequency range is 30—200 megacycles, \( C \) has a range of 10—60 microfarads, and the \( Q \) range is 80 to 1200.

The oscillator supplies a current \( I \) to \( R' \) and the unknown and \( C \). Since \( R' \) is very small compared with the impedance of the external circuit, the voltage \( E \) is equal to \( IR' \). Hence the reading \( I \) gives an indication of the voltage impressed on the circuit.

If the unknown and \( C \) are tuned to resonance (i.e., \( C \) adjusted until \( E_C = \text{Max.} \))

\[
I' = \frac{E}{R} \quad (1)
\]

where \( R \) is the effective series resistance of the unknown and \( C \).

Also

\[
E_C = I'X_C = \frac{E X_C}{R} = QE \quad (2)
\]

Hence if \( E \) is held constant by holding \( I \) constant, \( E_C \) reads directly proportional to \( Q \) and this meter may be calibrated directly in terms of \( Q \). Since \( E_C \) is proportional to \( E \), \( I \) may be calibrated in terms of a multiplying factor for extending the \( Q \) range of the instrument.

If the distributed capacity \( cd \) is appreciable in comparison with \( C \), the actual \( Q \) will be \( Q(1 + cd/C) \). This correction is usually negligible.

The above method is applicable for measuring inductance but not for measuring capacity or very low or high values of inductance. This difficulty may be overcome by connecting a coil \( L \) across terminals \( AB \), resonating the circuit, and reading \( C_1 \).
\(Q_1\), and the frequency \(f\). The unknown impedance \(Z\) is then connected in series with \(L\) as shown in Fig. 4-33(a), or in parallel with \(C\) as shown in Fig. 4-33(b).

\[
\begin{align*}
\text{a. Series Connection} & \quad \text{b. Shunt Connection} \\
\end{align*}
\]

\text{FIG. 4-33. MEASUREMENT OF IMPEDANCE.}

\(C\) is again adjusted for resonance and the new values \(C_2\) and \(Q_2\) are read. The constants of the circuit may then be calculated by means of the equations given in Table 4-1.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Test Circuit of Fig. 4-33a Results in terms of Fig. 4-34a</th>
<th>Test Circuit of Fig. 4-33b Results in terms of Fig. 4-34b</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Q)</td>
<td>(\frac{Q_1Q_2(C_1 - C_2)}{C_1Q_1 - C_2Q_2})</td>
<td>(\frac{Q_1Q_2(C_2 - C_1)}{C_1(Q_1 - Q_2)})</td>
</tr>
<tr>
<td>(R)</td>
<td>(1.59 \times 10^8 \frac{C_1Q_1 - C_2Q_2}{C_1C_2Q_1Q_2f})</td>
<td>(1.59 \times 10^8 \frac{Q_1Q_2}{C_1f(Q_1 - Q_2)})</td>
</tr>
<tr>
<td>(X)</td>
<td>(1.59 \times 10^8 \frac{(C_1 - C_2)}{C_1C_2f})</td>
<td>(1.59 \times 10^8 \frac{(C_2 - C_1)}{f})</td>
</tr>
<tr>
<td>(L)</td>
<td>(2.58 \times 10^{10} \frac{(C_1 - C_2)}{C_1C_2f})</td>
<td>(2.58 \times 10^{10} \frac{(C_2 - C_1)}{f^2})</td>
</tr>
<tr>
<td>(C)</td>
<td>(\frac{C_1C_2}{C_2 - C_1})</td>
<td>(C_1 - C_2)</td>
</tr>
</tbody>
</table>

If \(C_1 > C_2\), \(X\) is inductive
If \(C_1 < C_2\), \(X\) is capacitive

\(C_1\) and \(Q_1\) values with unknown removed; \(C_2\) and \(Q_2\) values with unknown in circuit. \(C\) in micro-microfarads, \(L\) in henries, \(f\) in cycles.

For simplicity in calculating, the values are given in terms of the equivalent series circuit of the unknown for the series-
connected test and in terms of the equivalent parallel circuit for the parallel-connected test.

Table 4–2 gives the equations that may be used to change the values from the equivalent series circuit to the values for the equivalent parallel circuit, or vice versa.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Series Circuit (Fig. 4–34a)</th>
<th>Parallel Circuit (Fig. 4–34b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R )</td>
<td>( \left( \frac{1}{1 + Q^2} \right) \times R \text{ Par} )</td>
<td>( (1 + Q^2) \times R \text{ Ser} )</td>
</tr>
<tr>
<td>( X )</td>
<td>( \left( \frac{Q^2}{1 + Q^2} \right) \times X \text{ Par} )</td>
<td>( \left( \frac{1 + Q^2}{Q^2} \right) \times X \text{ Ser} )</td>
</tr>
<tr>
<td>( L )</td>
<td>( \left( \frac{Q^2}{1 + Q^2} \right) \times L \text{ Par} )</td>
<td>( \left( \frac{Q^2}{1 + Q^2} \right) \times L \text{ Ser} )</td>
</tr>
<tr>
<td>( C )</td>
<td>( \left( \frac{1 + Q^2}{Q^2} \right) \times C \text{ Par} )</td>
<td>( \left( \frac{Q^2}{1 + Q^2} \right) \times C \text{ Ser} )</td>
</tr>
</tbody>
</table>

The method of arriving at the quantities in Table 4–2 may be illustrated by deriving the expression for the \( Q \) of a circuit as measured by the series connection method in Fig. 4–33(a). Let \( X_1 \) and \( R_1 \) represent the reactance and resistance of coil \( L \), and let \( X \) and \( R \) represent the reactance and resistance of the unknown impedance.

For the measurements with the unknown removed from the circuit

\[
Q_1 = \frac{X_1}{R_1} = \frac{1}{\omega C_1} \tag{3}
\]

and

\[
X_1 = \frac{1}{\omega C_1} \tag{4}
\]

Similarly with the unknown inserted in the circuit

\[
Q_2 = \frac{X_1 + X}{R_1 + R} = \frac{1}{\omega C_2 (R_1 + R)} \tag{5}
\]

and

\[
X_1 + X = \frac{1}{\omega C_2} \tag{6}
\]
where $X$ would be negative if it were capacitive. The $Q$ of the unknown will be

$$Q = \frac{X}{R} \quad (7)$$

Solving (5) for $R$ and (6) for $X$, we have

$$R = \frac{X_1 + X}{Q_2} - R_1 = \frac{1}{\omega C_2 L_2} - R_1 \quad (8)$$

and

$$X = \frac{1}{\omega C_2} - X_1 \quad (9)$$

Substituting in (5) gives

$$Q = \frac{\frac{1}{\omega C_2} - X_1}{1 - \frac{1}{\omega C_2 L_2} - R_1} = \frac{Q_2 - \omega C_2 L_2 X_1}{1 - \omega C_2 L_2 R_1} \quad (10)$$

But from (4) $X_1 = \frac{1}{\omega C_1}$, and from (3) $R_1 = \frac{1}{\omega C_1 L_1}$.

Hence

$$Q = \frac{Q_2 - \frac{C_2}{C_1} Q_1}{1 - \frac{C_2}{C_1} \frac{Q_2}{Q_1}} = \frac{Q_2 (C_1 - C_2)}{C_1 L_1 - C_2 L_2} \quad (11)$$

One limitation of the $Q$ meter is that the size of the various components must be such that the circuit can be tuned to resonance by means of $C$. This condition may usually be satisfied by the proper choice of $L$ and of the correct connection. For low-impedance circuits the series connection is usually preferred, and for high impedance the parallel connection is usually better.

The distributed capacity of a coil may be measured by the method shown in Fig. 4-32. The coil is connected to $AB$, $C$ is set to about 50 picofarads, and the frequency is adjusted to resonance. The frequency is then set at half the previous value and the circuit is adjusted to resonance by adjusting $C$ for resonance. The distributed capacity $C_d$ is then

$$C_d = \frac{C_2 - 4C_1}{3} \quad (12)$$

where $C_2$ and $C_1$ represent the two settings of $C$. 
The resonant frequency of a tuned circuit may be determined by loosely coupling this circuit to the inductance $L$ in the $Q$ meter circuit. The $Q$ of the $Q$ meter circuit will drop when the external circuit is tuned to the same frequency as that of the $Q$ meter circuit.

**Precautions**

1. The circuit must always be tuned to resonance by adjusting $C$ for maximum reading of the $Q$ meter.
2. All leads must be short and heavy.
3. The terminals $A$ and $B$ are not grounded; hence the unknown circuit should not be grounded except when the parallel connection is used.
4. The accuracy of the device for a very low or a very high $Q$ is definitely limited.
5. The coils $L$ must be shielded and precautions must be taken to prevent picking up extraneous signals.

**Procedure**

1. Measure the $Q$, $R$, and $I$ of a number of coils covering a wide range of inductances. Be sure to use both the series and parallel method.
2. Obtain data for and plot curves of $Q$ and $R$ against frequency over a wide range. Repeat this for several different types of coils.
3. Determine the effect of a coil shield on the $Q$ of the coil.
4. Measure the capacity and $Q$ of a number of condensers. Be sure to use both series and parallel methods.
5. Obtain data for and calculate the distributed capacity of a coil. Calculate the resonant frequency of the coil (i.e., frequency at which $C_d$ and the coil inductance are in resonance).
6. Measure the self-inductance of both windings of a radio frequency transformer. Measure the inductance of the windings connected series aiding and series opposing. Calculate $M$ from

$$L_{\text{series}} = L_1 + L_2 - 2M$$

**REFERENCES**

Boonton Radio Corp., Instructions and Manual of Radio-Frequency Measurements for $Q$ Meters, Type 100A 160A and 170A.

